Earth-like Planet in a Binary Star System

Introduction

Binary systems are not uncommon in the universe, and are found as systems of stars, planets, asteroids, and even black holes. Because the presence of an additional star creates more variation in the centerward mass distribution of a solar system, planets within binary star systems can have a number of interesting orbits. The two types of stable planetary orbits in such a system are called ‘S’ type, where the planet orbits only one of the binary stars, and ‘P’ type, where the planet orbits the entire system (Figure 1).

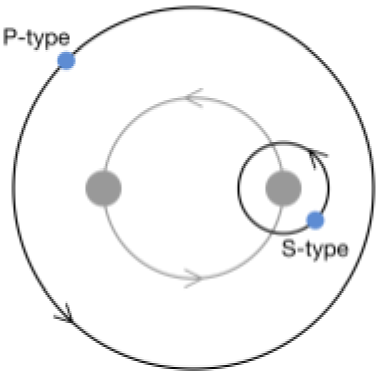


Figure 1: Visual of S and P type orbits

Exploring the possible orbits of a planet placed near a two-star system can result in interesting discoveries. There is a nearly infinite combination of star and planet masses and velocities, star separations, and planet starting positions, so there is always something new to discover in a new experimental setup. In the experiment to be presented within this report, the dynamics of a planet with the mass and velocity of the Earth in a binary star system is explored. The binary system was inspired by values from the Sirius binary system, simply to use some parameters from an existing system. In this setup, where the stars and planets always have the same initial mass and velocity magnitude, what planet behaviors will emerge at different starting points?

Procedure and Methods

Before proceeding with any actual simulations, several guidelines must be determined to ensure everything runs smoothly and accurately. To begin, units were needed. There are units that are commonly used in galactic studies, but they did not all work well in the modeling program, which is designed to work best with distances around the order of ~10^-2 to 10^2. Through a series of tests, it was determined that the best units for the program and simulations would be distances in astronomical units (AU), masses in solar masses ($M_\odot$), and time in days. In order to properly complete gravitational calculations in these units, the universal gravitational constant G was then recalculated with these units, as follows:

$$G = 4.003 * 10^{-3} \frac{pc}{M_\odot} \frac{km^2}{s^2} * \frac{3.086*10^{13} km}{1\ pc} * \frac{1\ AU^3}{(1.496*10^8 km)^3} * \frac{86400s}{day} = 2.9592 * 10^{-4} \frac{AU^3}{M_\odot * day}$$

With units chosen and G ready, the next requirement was to determine what variables to hold constant and which ones to vary and examine throughout the simulations. For this work, a small effort was made to prepare the system like something already seen in nature, so the constant parameters were inspired by existing bodies. For the planet, an Earth-like planet was used, with a mass equal to that of the Earth ($3.003 * 10^{-6} M_\odot$) and a matching velocity magnitude ($0.01733 \frac{AU}{day}$). To keep the velocity vector constant, starting velocity was chosen to always be in the counterclockwise direction, tangential to the star orbits. This is an arbitrary choice because testing with a variety of velocity vectors is beyond the scope of the method used to run the simulations, as well as beyond the scope of this project. The binary system was inspired by the Sirius binary, which contains one star of mass $2.063\,M_{\odot}$ and one of mass $1.018\,M_{\odot}$ (to be called, respectively, star 1 and star 2 from this point on). The actual Sirius binary has elliptical orbits, but for this project the distance was held constant at the smallest separation of the Sirius binary stars, 8.2 AU. Because of external interactions within the real system, as well as the choice to use circular orbits instead of the real elliptical shapes, the true velocity of these stars could not be used, and instead was calculated using the equations

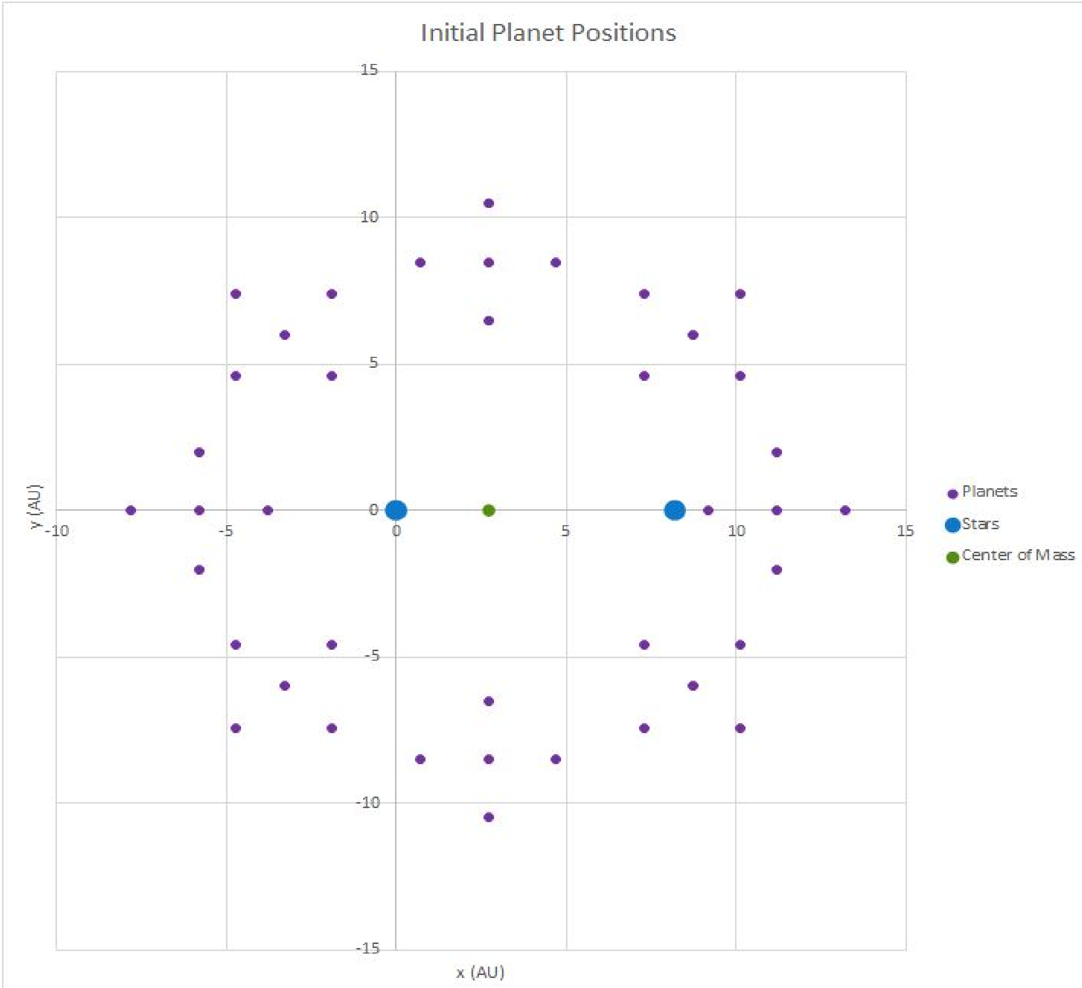
$$\frac{m_1 v_1^2}{r_1} = G\frac{m_1 m_2}{r_{total}^2} \tag{1}$$

$$v = \sqrt{G\frac{r_1 m_2}{r_{total}^2}} \tag{2}$$

From the center of mass (COM = 2.7094 AU), star 1 has an orbital radius of 2.7094 AU, star 2 has an orbital radius of 5.4906 AU. This results in a velocity of $3.4840 * 10^{-3}\frac{AU}{day}$ for star one and $7.0605 * 10^{-3}\frac{AU}{day}$ for star two. Note that rounding numbers will result in imperfect initial velocities, so this system may drift slightly, but not enough that it will present a concern for the simulations.

The set of initial conditions was designed to cover a considerable area of the parameter space without a large number of simulations. To do this, a compass-like spread of points was created, with a center at the stars' center of mass in their initial positions. These star initial positions were constant throughout all the simulations. The planet points were aligned and placed just outside the largest orbit radius of the two stars to avoid immediate collisions. At each directional point of the compass, five test points were placed in a '+' shape, spaced evenly on a 2x2 AU grid. The closest point to the center of mass of each '+' was placed 1 AU from the largest orbit, simulating the distance between the Earth and Sun in our solar system. It is of course not expected that this planet would behave as the Earth normally does, but this choice provides a starting point for the simulations. The resulting spread of initial conditions involves 40 starting points surrounding the star orbits. The visual of this layout is below in Figure 2, in which the groups will now be referred to as groups 1-8, starting on the left and moving clockwise:

Figure 2 (below): All initial planet positions for the 40 simulations. Purple dots represent each planet, the blue dots are the stars (1 on left, 2 on right), and the green dot is the center of mass. The planet locations are spread equally around the center of mass in 8 groups of 5.



which becomes quantitatively (in AU):

x	y	x	y	x	y	x	y
-3.7812	0	-1.88	4.59	2.709	6.491	7.299	4.59
-5.7812	0	-3.294	6.004	2.709	8.491	8.713	6.004
-5.7812	-2	-4.709	7.418	2.709	10.491	10.127	7.418
-5.7812	2	-1.88	7.418	4.709	8.491	7.299	7.418
-7.7812	0	-4.709	4.59	0.709	8.491	10.127	4.59
0	-0.01733	-0.01225	-0.01225	-0.01733	0	-0.01225	0.01225

x	y	x	y	x	y	x	y
9.2	0	7.299	-4.59	2.709	-6.491	-1.88	-4.59
11.2	0	8.713	-6.004	2.709	-8.491	-3.294	-6.004
11.2	-2	10.127	-7.418	2.709	-10.491	-4.709	-7.418
11.2	2	10.127	-4.59	0.709	-8.491	-1.88	-7.418
13.2	0	7.299	-7.418	4.709	-8.491	-4.708	-4.59
0	0.01733	0.01225	0.01225	0.01733	0	0.01225	-0.01225

The bottom row of each table is the velocity of each planet in that set, in units of AU/day.

At last, with all initial conditions, parameters, and constants determined, the simulation code was created. This code uses the Verlet algorithm, and was adapted from code that originally calculated dynamics of a Leonard Jones droplet. It was modified for gravitational dynamics by introducing the gravitational equations below (3 and 4), in place of the respective Leonard Jones force and energy equations.

$$F = G\frac{m_1 m_2}{r_{total}^2} \qquad (3)$$

$$U = -G\frac{m_1 m_2}{r_{total}} \qquad (4)$$

During the force calculation, a check was included on the distance between bodies. If the distance vector becomes larger than 50 AU or smaller than 0.1 AU, the simulation is automatically ended. This prevents unnecessary calculations if the planet leaves the system or collides with a star, and prints to the screen the reason for ending. To ensure proper function of the program, several test simulations were completed, some of which only included stars, because their expected behavior is well known and any error would be very clear in an animation of the results. These tests also determined that the simulations have a nearly constant total energy, a good indicator of accuracy in the code. With everything in place, separate folders were created and a simulation was run for every initial planet condition.

Data and Results

Overall, most planets in the simulation did not show significant interaction with the stars, and therefore they did not survive in the system for long. Of the 40 simulations run, 87% escaped with little to no interaction. This means that 35 of the initial positions chosen did not demonstrate any type of potential orbit or other interesting behavior, but they still provide insight to the possible behaviors of planets put into this system. The remaining 5 simulations were unique, and one simulation developed into a stable orbit on the timescale of 1000 years.

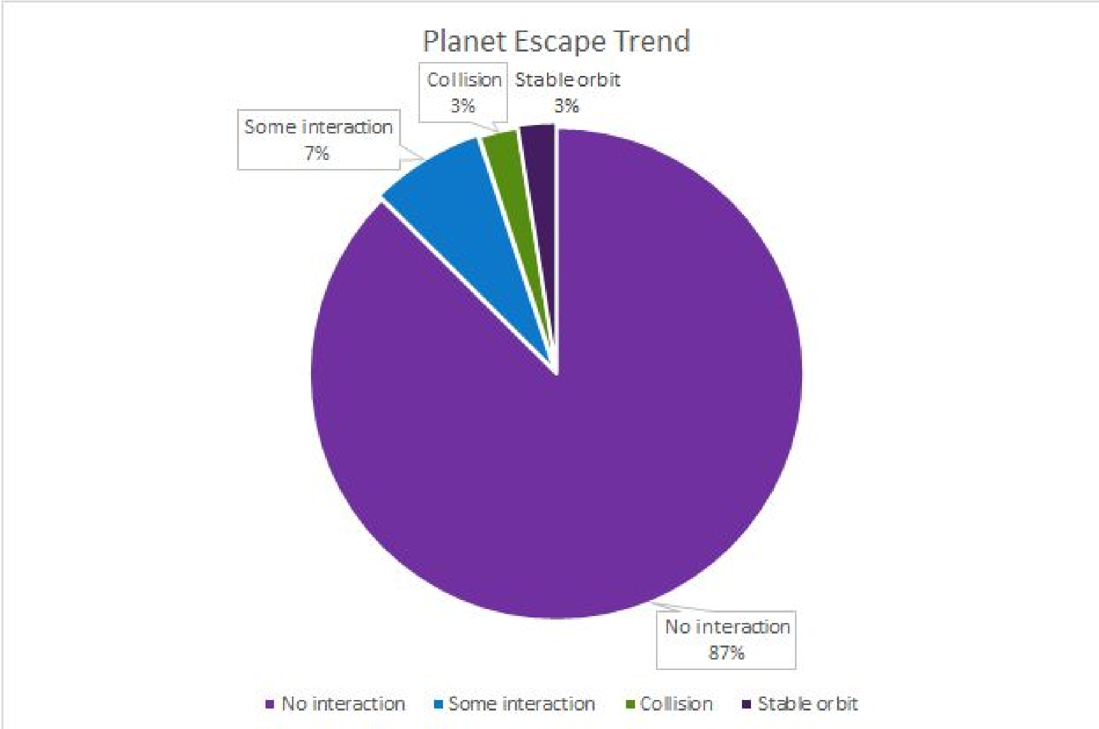


Figure 3: A survey of the escape trends found in the 40 planet positions.

Figure 3 above shows the escape trends of the planet positions. No interaction means that the planet effectively went from its initial position straight out of the system, with no pull from the stars strong enough to make the planet take a curved path. Some interaction means the planet was pulled into the binary system and looped around the stars for some period of time, often in a figure-8 shape. The single collision and stable orbit planets are explored at the end of this section.

Because most of the planets experienced no significant gravitational pull from the stars, most planets also escaped the system quickly, around the same time frame. The range of escape times is shown in the first plot below (Figure 4), in which the majority of planets escape between 5 and 15 years. This plot excludes the 1000 year surviving planet of group 5, because otherwise significant detail would be lost. A careful observation of this plot shows that planets that were near a star early in the timeline (such as those in groups 1, 4, 5, and 8) survive longer. Meanwhile, planets that were not near a star early on escaped very quickly. This is a behavior that is expected, because gravitational pull of the stars on the planets is what keeps them in orbit. For planets that do not experience a nearly immediate gravitational pull, their velocity puts them on a straight path away from the binary with no hope of recapture. These observations are shown very clearly in the second plot below this paragraph (Figure 5), which shows the average time of all planets in a group to escape. As expected, groups 1, 4, 5, and 8 have the longest survival times. This graph does show some skewing of data, because it is often the case that only one planet survived past the common 10 year life. However, it still clearly points to the groups with desirable initial positions for capture, even if only one planet was actually in that position.

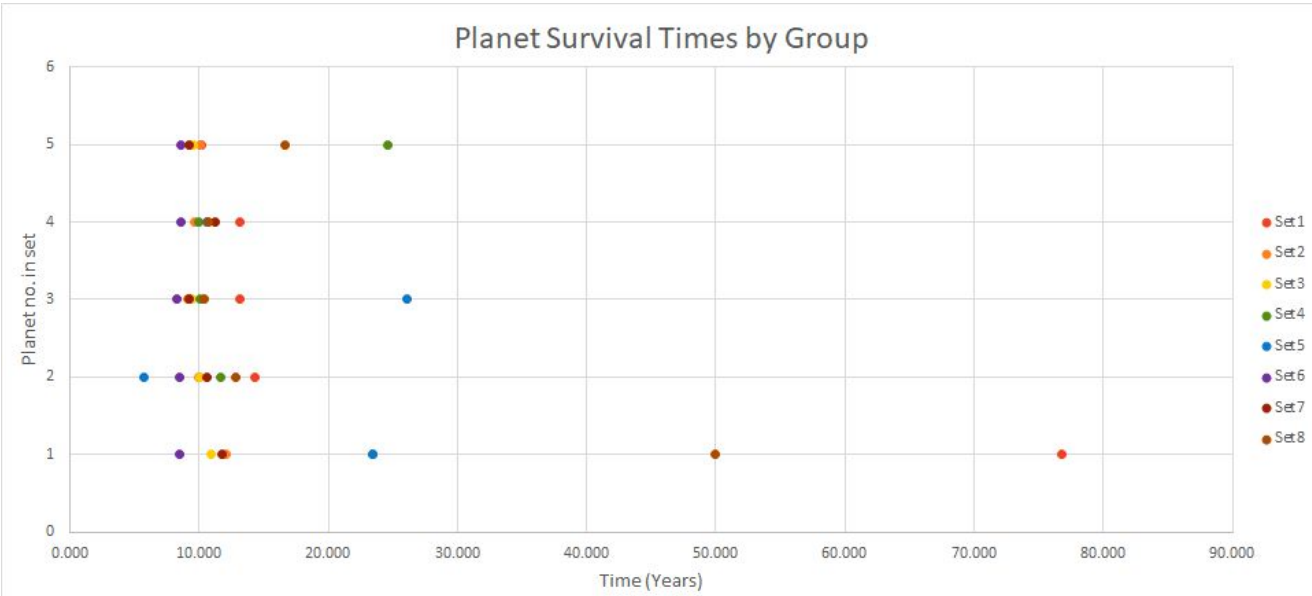


Figure 4: Timeline of all planet escapes, excluding the 1000 year survival planet from set 5. Most planets escape within the first 15 years; the longer lasting outliers belong to groups that start near the stars.

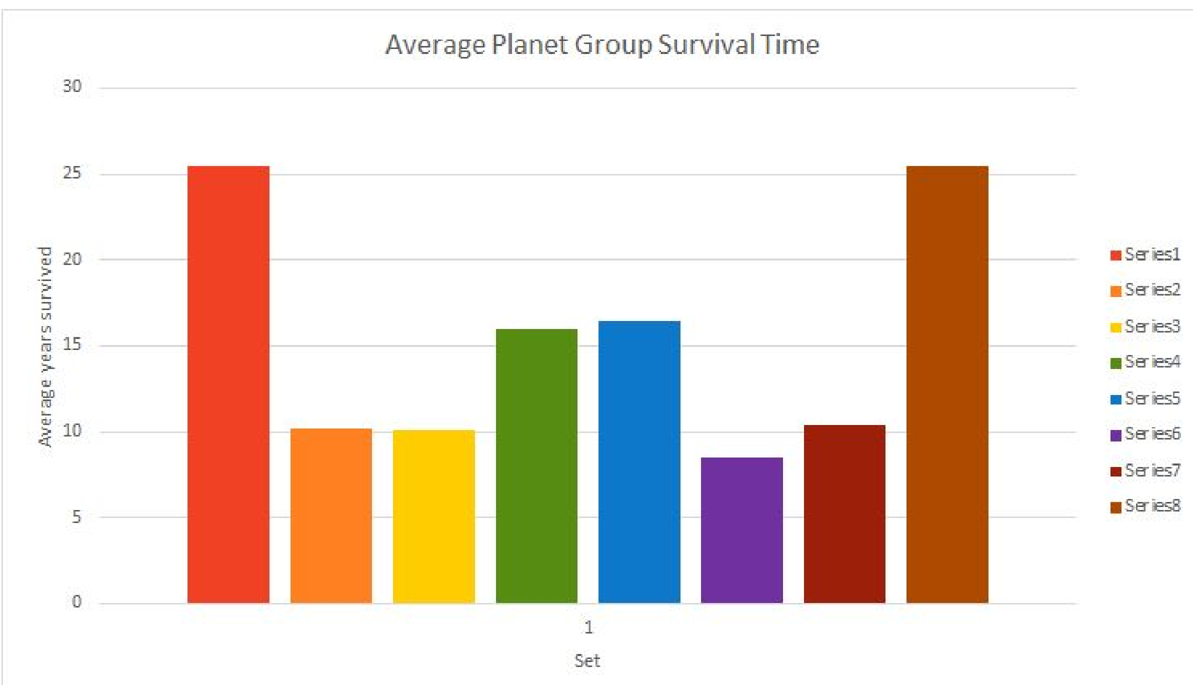


Figure 5: Average survival time of each planet group. The 1000 year outlier of set 5 is not included. Note that the longest surviving groups are the two which are closest to the heaviest star at the star of the simulation.

Of the 5 planets that showed unique behavior, two stand out. These are the planets that did not escape the system, but instead one survived the entire 1000 year time, and the other collided with the large star fairly early on. Both of these planets were part of group 5, which is the set of simulations that started out closest to either star. Their trajectories prove that planets beginning closer to the stars will have considerable interactions. If this were a case where the planet approached from outside the system, the behavior of group 5 demonstrates that the planet would have to pass very near to the stars for there to be any chance of a capture in the system.

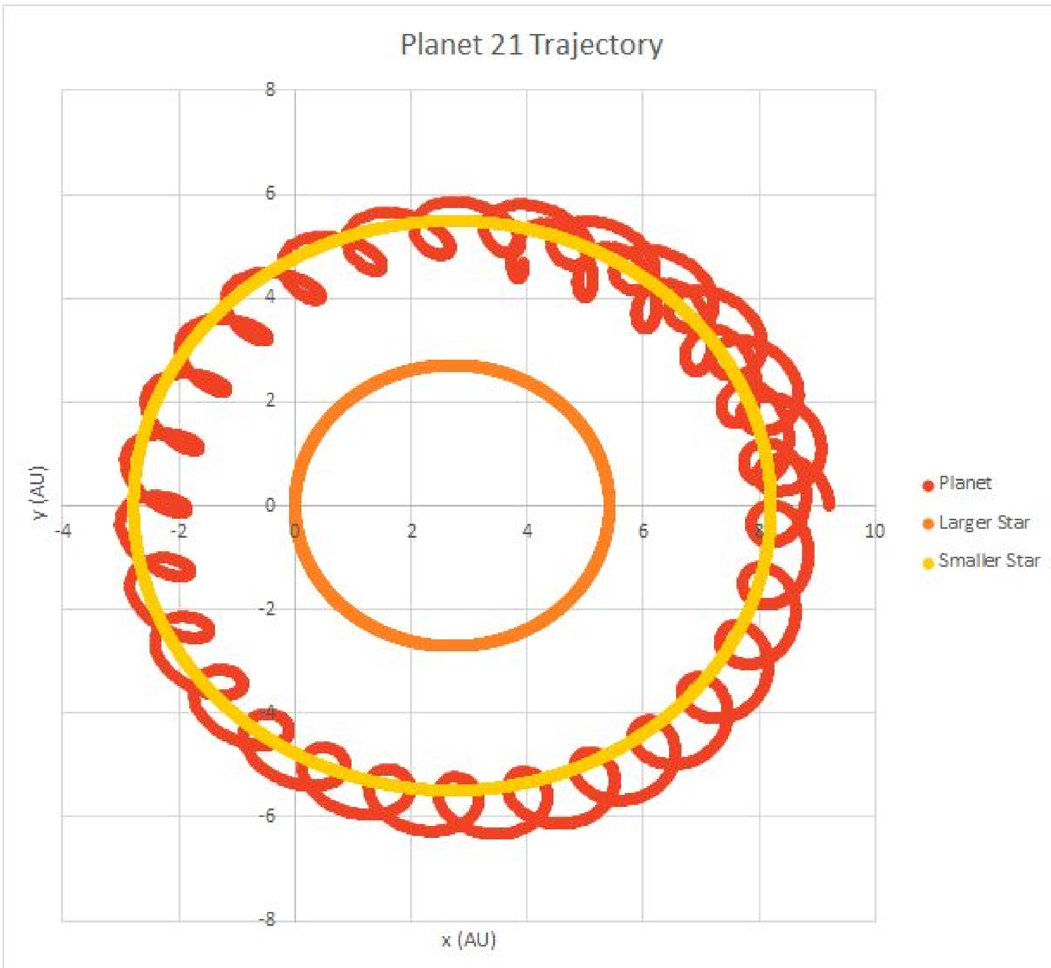


Figure 6: Trajectory of planet 21, which had initial conditions starting it closest to a star than any other planet in the simulation set. It is pulled into an elliptical orbit around the smaller, Sun-like star.

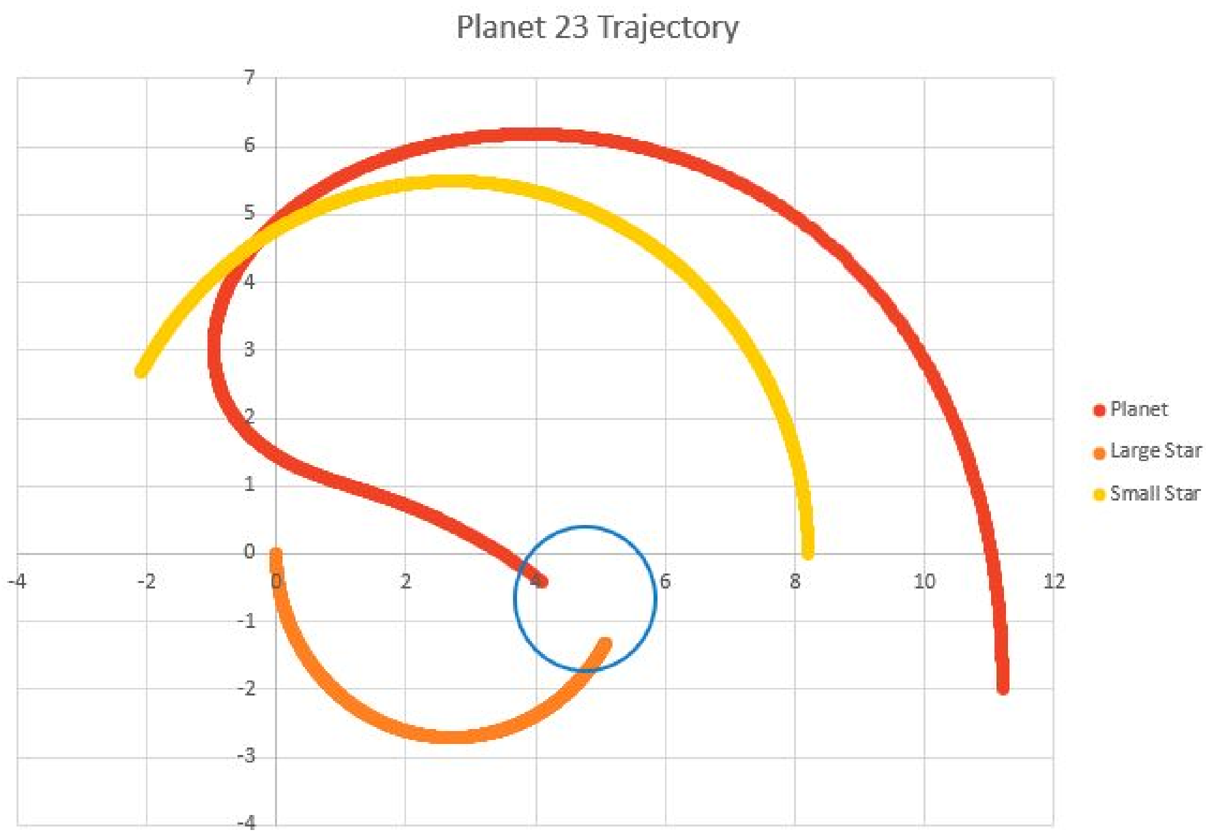


Figure 7: Trajectory plot of planet 23, which collided with the larger star at the location of the blue circle. This plot shows the significance of starting location. The planet began just near enough to the small star to be pulled into a curved path, but got too close to the star which put the planet into a path towards the center of mass and directly into the larger star.

Discussion and Conclusions

As shown by the high amount of planets in the simulations that had minimal interaction with the stars, there are very few and specific locations surrounding this binary system that support capture of the Earth-like planet. The primary reason for this is proximity requirements. It is of course known that an Earth-like planet must be 1 AU from a 1 $M_\odot$ star if the dynamics are to work out into an orbit. Planet 21 proved this again, as it began 1 AU from the small star and moved into a stable orbit very quickly. The rest of the planets began too far from the stars to be captured in an orbit, demonstrating that (at the planet velocity and mass used), the planet must cross very close to a star at some point in order to be captured by the system. This means that if the planet were an approaching body, the alignment of the stars at the time it passes would be a crucial factor in the possibility of planet capture.

The simplistic nature of these simulations lead to a few issues, or areas where improvements would be ideal in a future examination of this star and planet system. First, there seems to be a small error within the energy calculation sections of the code. Its cause was not determined because the trajectories of the planets were accurate, and the energies were not an important part of the analysis done. However, this is still a problem and would need to be fixed in the future.

The second problem is not necessarily an issue, but something that could be improved upon. Because each simulation had to be run by hand, so the set of planets had to be small. This significantly limited amount of variety that could be placed into the simulation parameters. There is a lot that could be discovered in using other parameters, such as experimenting with a planet approaching from outside of the system, which would be more realistic than placing a planet directly next to a star at the beginning of a simulation run. There is a list of ways to make running large amounts of simulations possible with the code in this lab, such as a bash script to run the simulations with one command, then using a graphing program and code to simplify the simulation output. Python's matplotlib module would be appropriate for this, as one example.

Ultimately, the simulations run and examined for this lab provide numerous conclusions for the nature of binary star systems, but there are still infinite situations that could also be explored within the system. Determining a range of distances and locations to create S and P type orbits in the system, for example, could produce fascinating results and provide visuals to help teach people about the nature of binary systems.

References

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2. "Sirius." *Wikipedia*, Wikimedia Foundation, 17 Apr. 2020, en.wikipedia.org/wiki/Sirius.